Optimum Design of Composite Structures with Stress and Displacement Constraints

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Theme

A METHOD based on an optimality criterion is presented for designing minimum weight fiber reinforced composite structures with stress and displacement constraints. A recurrence relation based on an optimality criterion is used to modify the design variables during the iterative procedure. The displacement method of finite element analysis is used to determine the response of the structure to the applied loads. The method takes into consideration the effects of multiple displacement constraints, multiple loading conditions, and variable allowable stresses. Illustrative examples are given to show the application of the method to structures with certain displacements and twist specified, a problem basic to the developing technology of aeroelastic tailoring.

Contents

The subject of optimization in general and the automated design of structures in particular has received wide attention in recent years. The methods based on optimality criteria are found to be very efficient in designing structures with a large number of design variables. The overall review of the optimality criteria methods is given in Refs. 1 and 2.

In recent years considerable attention has been focused on controling the deflected shape of a wing because of the new concept of Control Configured Structures. The basic idea is to control the deflection, twist and camber under the applied loads to favorably influence the aerodynamic forces. The use of fiber reinforced composites in structures is most suitable for controlling the deflected shape, due to their different elastic properties. The effectiveness of elastic properties in different directions can be changed by a suitable selection of fiber orientations and the number of laminae in these fiber directions. The general practice in aerospace industry is to select a certain number of fiber directions and change the number of laminae in these fiber directions. For illustration here a laminate with fibers in 0°, 90°, ±45° is used. The selection of these four fiber directions is arbitrary, and it should not be considered as a requirement of the proposed method.

The weight of the structure is given by

$$W=\sum_{i=1}^{m}\sum_{j=1}^{n}\rho_{ij}A_{ij}\ell_i \tag{1}$$

where ρ_{ij} is the density, and A_{ij} is the thickness of the jth layer of the ith element. ℓ_i is the area of the ith element. m is the number of elements in the structure, and n is the number of layers in the composite elements. When the structure has displacement constraints ($\bar{C}_p$) at P locations, the constraint equation is

$$\{r\}^t\{S^P\}-\bar{C}_p=C_p-\bar{C}_p=0 \quad p=1,2,\text{---}P \tag{2}$$

where $\{r\}^t$ is the displacement vector due to the applied load, $\{S^P\}$ is the virtual load vector consisting of a unit load in the direction of pth constraint and zero forces in all other directions. C_p is the actual displacement at the pth constraint. Using Eqs. (1) and (2) the lagrangian $\bar{F}$ can be written as

$$\bar{F}=\sum_{i=1}^{m}\sum_{j=1}^{n}\rho_{ij}A_{ij}\ell_i+\sum_{p=1}^{P}\lambda_p[\{r\}^t\{S^P\}-\bar{C}_p] \tag{3}$$

where λ_p are the positive Lagrangian multipliers. For a necessary condition of optimality, the partial derivatives of Eq. (3) with respect to the design variables A_{ij} should be zero. Applying this condition and rearranging the terms, the optimality conditions can be written as

$$1=e_{ij}^P/\rho_{ij} \tag{4}$$

where

$$e_{ij}^P=\{r\}_i^t[k]_{ij}\left\{\sum_{p=1}^{P}\lambda_p\{s^p\}_i\right\} \tag{5}$$

where $\{r\}_i$ and $\{s^p\}_i$ are the displacement vectors associated with the ith element and $[k]_{ij}$ is the stiffness matrix of the jth layer of the ith element. The recurrence relation can be written as

$$(\alpha_{ij}\Lambda)_{\nu+1}=B(\alpha_{ij})_\nu\left[\frac{\bar{e}_{ij}^P}{\rho_{ij}}\right]_\nu^a \tag{6}$$

where ν and $\nu+1$ refer to the iteration number, and α_{ij} is the relative design variable such the $\alpha_{ij}\Lambda=A_{ij}$. In Eq. (6), $\bar{e}_{ij}^P$ is obtained from Eq. (5) by replacing all quantities by their relative values. In Eq. (6) the parameter 'a' determines the step size. $a=½$ is considered as a normal step size and is used in the illustrative problems. To use Eq. (6) it is necessary to know the relative values of the Lagrangian parameters, $\tilde{\lambda}_P$. $\tilde{\lambda}_P$ should be zero for nonactive constraints. In an iterative procedure the active and nonactive constraints may change from one iteration to another. The selected values of $\tilde{\lambda}_P$ should take into consideration the relative importance of each specified constraint. A recurrence formula for $\tilde{\lambda}_P$ can be written by using the constraint Eq. (2) ($C_P=\bar{C}_P$). Multiplying this Equation by $\tilde{\lambda}_P^{1/b}$ and rearranging, the recurrence relation can be written as

$$\tilde{\lambda}_p^{\nu+1}=(C_p/\bar{C}_p)^b\tilde{\lambda}_p^\nu \tag{7}$$

where $(C_P/\bar{C}_P)^b$ is the amplification factor to readjust $\tilde{\lambda}_P$ based on its performance during the νth iteration.

In the case of a stress constraint design, when the allowable stresses are the same in all members of the structure, a

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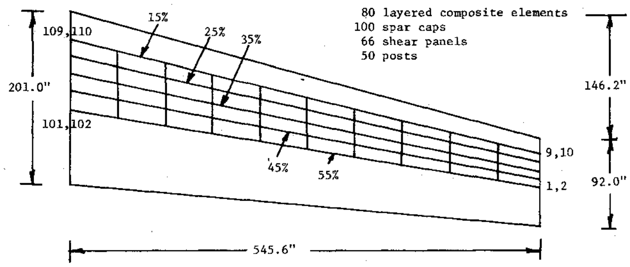

Fig. 1 Wing.

recurrence relation formulated on the basis of an optimality criteria, derived to satisfy the general stiffness of the structures can be satisfactorily used to design a minimum weight structure.[3] The optimality criteria can be expressed as

$$1=\lambda \frac{\bar{e}_{ij}}{\rho_{ij}} \text{ where } \bar{e}_{ij} \frac{\{r\}^t_i[k]_{ij}\{r\}_i}{A_{ij}\ell_i} \tag{8}$$

In the paper a recurrence relation with a parameter to adjust the step size is proposed as an approximation for variable stress limit problems. The recurrence relation is

$$(\alpha_{ij}\Lambda)_{\nu+1}=\bar{B}(\alpha_{ij})_\nu\left[\frac{\bar{e}'_{ij}}{\rho_{ij}}\beta_{ij}\right]^{1/2}_\nu \tag{9}$$

where $\bar{B}$ is a constant, and β_{ij} is a new parameter which is changed at each iteration. The recurrence relation for β_{ij} is given by

$$(\beta_{ij})_{\nu+1}=(\beta_{ij})_\nu\left[\frac{\bar{e}'_{ij}}{\tau_{ij}}\right]^b_\nu \tag{10}$$

where b is a parameter which adjusts the step size. $\bar{e}'_{ij}$ in Eqs. (9) and (10) is obtained from Eq. (8) by replacing all quantities by their relative values. τ_{ij} is the maximum strain energy density in a subelement, when it is subjected to a limiting stress in the direction of the fibers. For multiple loading conditions, recurrence relations are given in the paper.

A finite element computer program was written incorporating the various iteration formulas. The program has provision for three strength criteria which are normally used to design composite structures.

The plan form of the wing is shown in Fig. 1. The structural box consists of spars, ribs and coverplates. The top and bottom skin of the wing are represented by 80 composite elements consisting of three layers with fiber orientations in 0°, ±45° directions. The 0° fibers are parallel to the 50% chord. The max stress criteria was used for this problem. Loads, dimensions etc. are given in the paper.

The structure is designed by using Eq. (9) for the stress constraint problem. The initial weight of the structure is 5358.51 lbs. For this design the percentage of laminae in the 3 fiber direction is equal, and the relative size for the posts is 1.0 and for all other elements is 0.1. In using Eq. (9) 3 cases are considered. In the case I $\beta_{ij}=1$ so that the members are sized only on the basis of the relative strain energy density $\bar{e}'_{ij}$ and the mass density ρ_{ij}. In case II, $\beta_{ij}=1/\tau_{ij}$, and for the third case β_{ij} is changed according to Eq. (10). In Table 1 the minimum weights obtained for the various cases are given. The lowest weight is 940.74 lb for case III with $b=0.04$. The number of iterations required to attain the minimum weight design is also given in Table 1. The deflections in the z direction for the minimum weight design are 52.36", 52.36", 51.48", and 51.47" at nodes 1, 2, 9, and 10, respectively. In the displacement constraint problem, the deflection in the z direction at nodes (1,2) and (9,10) is restricted to 41.0" and 41.5", respectively. The purpose of this problem is to show how the displacement constraint algorithm can be used to reduce the top deflections and reverse the twist pattern. When the initial value of $\bar{\lambda}_P$ in Eq. (7) is equal to unity is used for all the four nodes, the rate of convergence is slow. Therefore $\bar{\lambda}_P$ equal to 1.0 and 0.15 was selected for nodes (1,2) and (9,10) respectively. The iteration history is given Table 2. The deflections at any one node, after the first iteration, are not equal to the specified displacements because during the scaling procedure the design is governed by the stresses in the numbers. After the first 4 iterations the twist pattern is reversed, and subsequent iterations are primarily for achieving the specified twist rather than reducing the weight of the structures. More illustrative examples are given in the paper.

Table 1 Stress constraint design

Case	Value of h	Wt (lbs)	Iter. no.
I		1044.77	20
II		979.63	22
III	1.0	1023.98	11
	0.5	1000.55	13
	0.05	948.00	21
	0.04	940.74	21
	0.01	981.68	27

Table 2 Displacement constraint design

Iteration no.	Weight lbs.	Displacement in z direction at nodes 1	2	9	10
1	5521.4	41.00	40.99	40.00	40.00
2	2320.7	40.99	40.92	40.30	40.30
3	1775.1	40.97	40.89	40.85	40.84
4	1472.6	40.98	40.91	41.18	41.17
5	1301.3	40.99	40.92	41.40	41.39
7	1149.4	40.46	40.38	41.39	41.39
10	1117.6	40.94	40.87	41.12	41.11
20	1069.6	40.98	40.91	41.34	41.33
40	1055.5	40.90	40.84	41.38	41.38
45	1053.2	40.94	40.89	41.46	41.45

The design algorithms based on optimality criteria give the best results when the problems either have displacement constraints or uniform stress constraints. When there are both active displacement constraints and stress constraints, various generalizations have to be introduced. These generalizations are equivalent to step size adjustments. Some of these generalizations are discussed in Refs. 1, 2 and in the original paper.

References

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